G. S. Mandal's
Maharashtra Institute of Technology, Aurangabad
(An Autonomous Institute)
Department of Mechanical Engineering

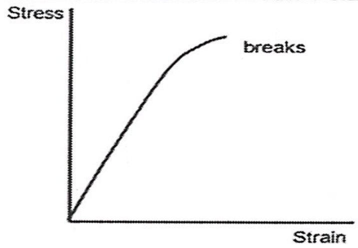
END SEMESTER EXAMINATION

Academic Year 2022-23 Semester-I

Class: Second Year **Date: 10/1/2023**

Course: Strength of Materials (MED-201) **Time: 2 hrs**

Max Marks: 50

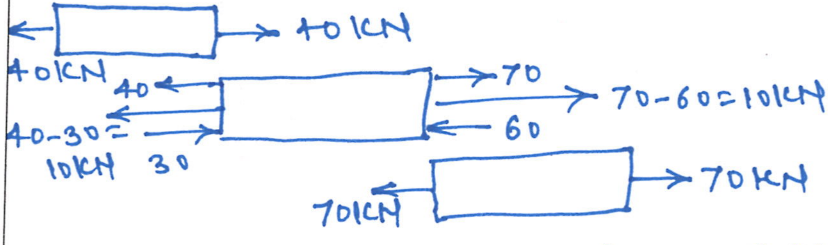
Q. 1	**Solve/Answer the following questions**	
	Answer/Solution	**Stepwise Marks**
a)	**Stress:** Internal resisting force per unit cross-sectional area is called stress. OR It is defined as the ratio of force per unit cross-sectional area. Unit: N/mm^2, N/m^2, and Pascal (Pa)	2
b)	 Stress-strain curve for brittle material	2
c)	**Classification of beams:** • Simply supported beam • Cantilever beam • Overhanging beam • Continuous beam • Fixed beam	2
d)	**Circumferential Stress:** The stress which act in tangential direction to the circumference of the cylinder due to internal pressure of fluid is called circumferential stress. **Longitudinal Stress:** The stress which act in longitudinal (axial) direction of the cylinder due to internal pressure of fluid is called longitudinal stress.	1 1
e)	**Core or Kernel of a Section:** The centrally located portion of the column section within which the load must act so as to produce only compressive stress in the column section is called core or kernel of a section.	2
f)	**Principal Plane:** The plane which carry only normal stress and no shear stress is called principal plane. The principal planes are always right angles to each other.	2
g)	**Torsional Stiffness:** It is the ratio of torque per unit angular displacement of the shaft. It is denoted by k_t. Mathematically, $k_t = \frac{T}{\theta} = \frac{G\,I_p}{L}$ Its unit is Nm/radians	2
h)	**Proof Resilience:** The maximum strain energy stored in the body without causing permanent deformation (i.e. up to elastic limit) is known as proof resilience. Mathematically, $$U_{max} = \frac{(\sigma_{max})^2}{2E} V$$	2

Q. 2	**Solve any one of the Following.**	
a)		

$A_1 = \frac{\pi}{4}\times 30^2 = 706.85\,mm^2$ $\quad A_2 = \frac{\pi}{4}\times 60^2 = 2827.4\,mm^2$ — 3M

$A_3 = \frac{\pi}{4}\times 40^2 = 1256.63\,mm^2$

$\Sigma F_x = 0,\quad -40+30-P+70=0,\quad P = 60\,kN$ — 1M



— 2M

$P_1 = 40kN\,(T),\quad P_2 = 10kN\,(T),\quad P_3 = 70kN\,(T)$

$\delta L = \delta L_1 + \delta L_2 + \delta L_3$

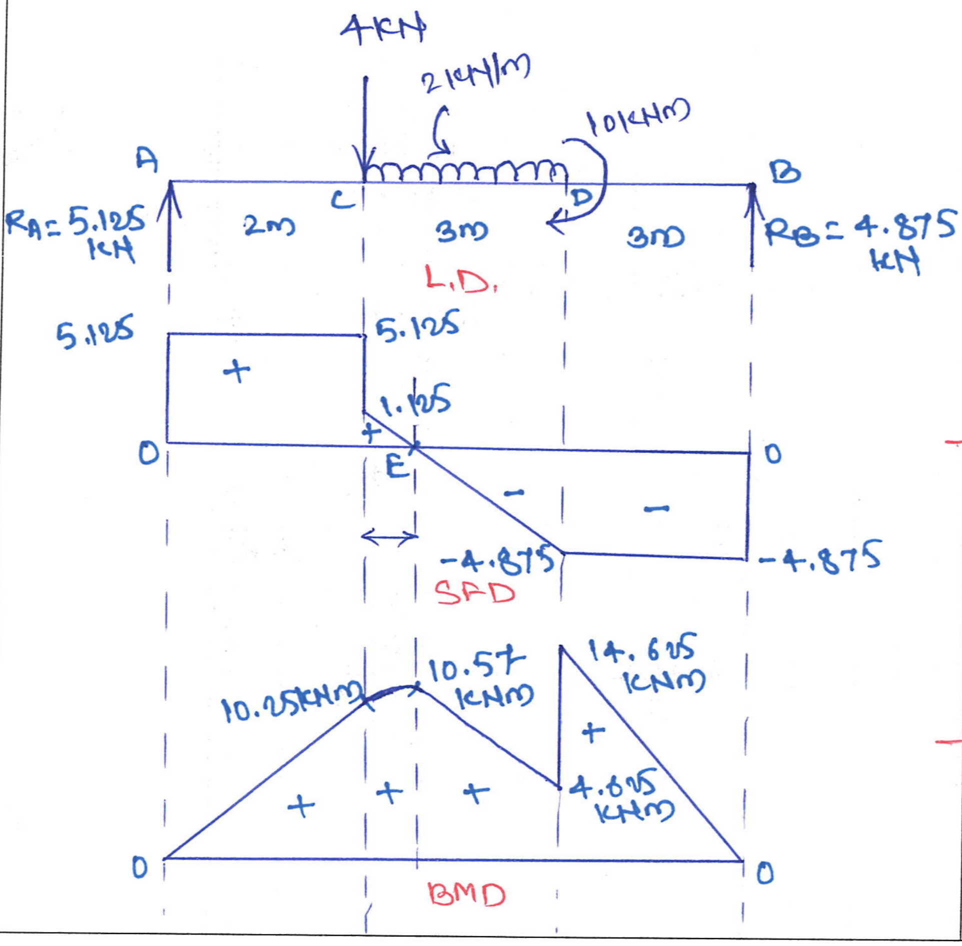
$\delta L = \frac{P_1L_1}{A_1E_1} + \frac{P_2L_2}{A_2E_2} + \frac{P_3L_3}{A_3E_3}$ $\quad \{E = const.\}$

$\delta L = \frac{1}{E}\left[\frac{P_1L_1}{A_1} + \frac{P_2L_2}{A_2} + \frac{P_3L_3}{A_3}\right]$

$\delta L = \frac{1}{202\times10^3}\left[\frac{40\times10^3\times600}{706.85} + \frac{10\times10^3\times800}{2827.4} + \frac{70\times10^3\times900}{1256.63}\right]$

$\delta L = 0.430\,mm\ (Ext.)$ — 2M

b)



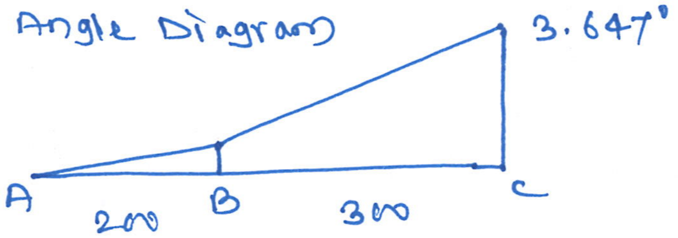
— 2M (RA,RB)

— 2M calculation, 1M Diagram

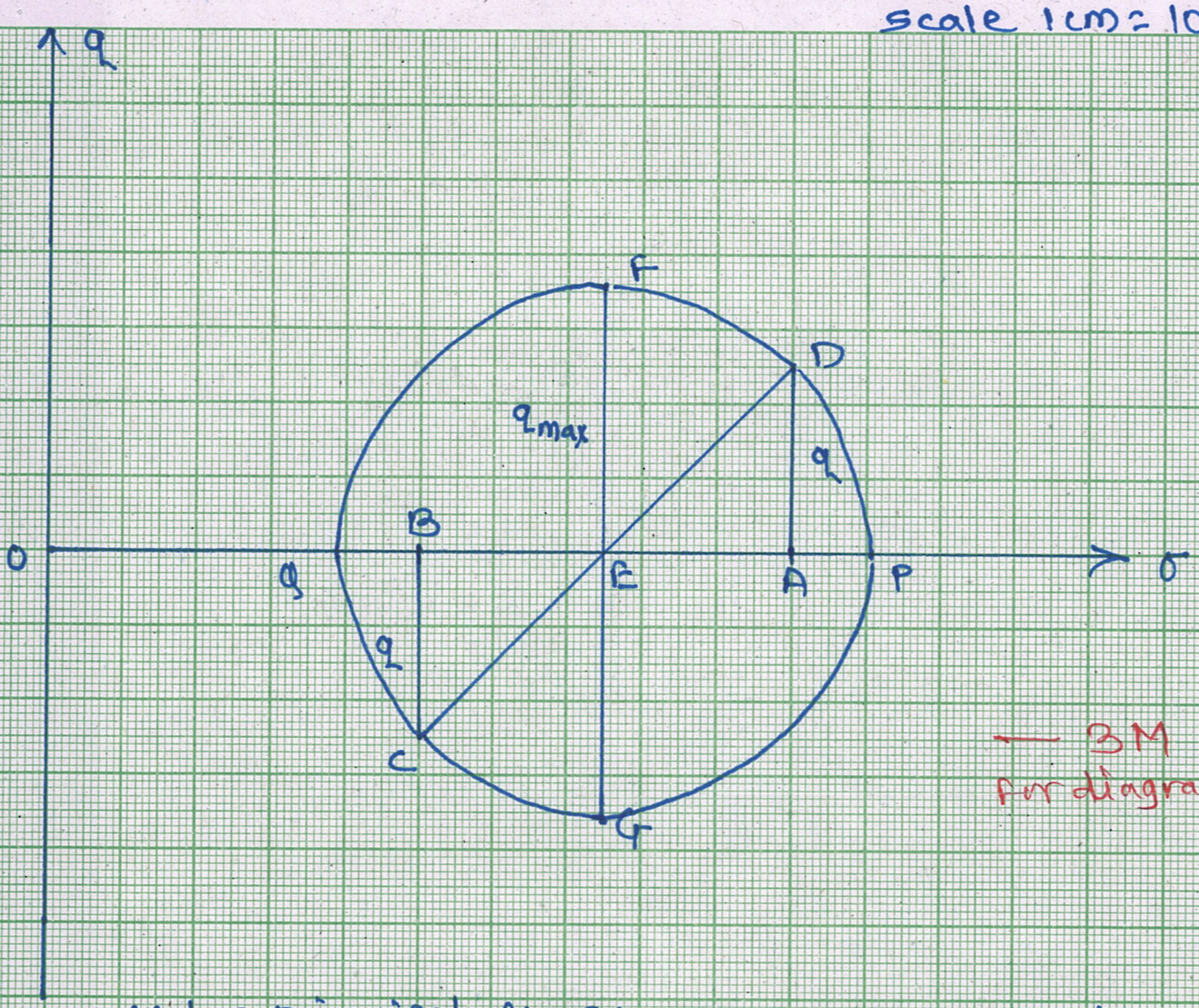
— 2M calculation, 1M Diagram

Q. 3	$L = 2m = 2\times10^3 mm$, $d = 1m = 1\times10^3 mm$ $t = 20mm$, $P = 12\ N/mm^2$, $E = 210\ GPa$, $\mu = 0.30$	
	$\sigma_c = \frac{Pd}{2t} = \frac{12\times1\times10^3}{2\times20} = 300\ N/mm^2$	— 1M
	$\sigma_L = \frac{Pd}{4t} = \frac{12\times1\times10^3}{4\times20} = 150\ N/mm^2$	— 1M
	$e_c = \frac{\delta d}{d} = \frac{\sigma_c}{E} - \mu.\frac{\sigma_L}{E}$ OR $= \frac{Pd}{4tE}(2-\mu)$	
	$e_c = 1.21\times10^{-3}$, $\delta d = e_c\times d = 1.21\ mm$	— 1M
	$e_L = \frac{\delta L}{L} = \frac{\sigma_L}{E} - \mu.\frac{\sigma_c}{E}$ OR $= \frac{Pd}{4tE}(1-2\mu)$	
	$\delta L = 2.857\times10^{-4}$, $\delta L = e_L\times L = 0.571\ mm$	— 1M
	$e_T = \frac{\delta T}{T} = -\mu.\frac{\sigma_c}{E} - \mu.\frac{\sigma_L}{E}$ OR $= -3\mu\frac{Pd}{4tE}$	
	$e_T = -6.428\times10^{-4}$, $\delta T = e_T\times T = -0.0128\ mm$	— 1M
	$e_v = e_L + 2e_c$ OR $= \frac{Pd}{4tE}(5-4\mu)$	
	$e_v = 2.705\times10^{-3}$	— 1M
	$e_v = \frac{\delta V}{V}$, $V = \frac{\pi}{4}\times d^2\times L = 1.57\times10^9 mm^3$	— 1M
	$\delta V = e_v\times V = 4.249\times10^6\ mm^3$	— 1M

Q. 4	$b = 4m = 4\times10^3$ mm $d = 3m = 3\times10^3$ mm $P = 600kN = 600\times10^3$ N, $e_x = 0.5m = 500mm$ $e_y = 1m = 1000mm$ $A = b\times d = 4\times10^3 \times 3\times10^3 = 12\times10^6$ mm^2	— 1M
	$I_{xx} = \frac{bd^3}{12} = \frac{4000\times3000^3}{12} = 9\times10^{12}$ mm^4 $I_{yy} = \frac{db^3}{12} = \frac{3000\times4000^3}{12} = 1.6\times10^{13}$ mm^4	
	$X = \frac{b}{2} = \frac{4000}{2} = 2000$ mm	— 1/2M
	$Y = \frac{d}{2} = \frac{3000}{2} = 1500$ mm	— 1/2M
	$\sigma_0 = \frac{P}{A} = \frac{600\times10^3}{12\times10^6} = 0.05$ N/mm^2	— 1M
	$\sigma_{bx} = \frac{M_x}{Z_{xx}} = \frac{P\times e_x}{I_{xx}} \times Y = 0.05$ N/mm^2	— 1M
	$\sigma_{by} = \frac{M_y}{Z_{yy}} = \frac{P\times e_y}{I_{yy}} \times X = 0.075$ N/mm^2	— 1M
	stress at each corner $\sigma_A = \sigma_0 + \sigma_{bx} - \sigma_{by} = 0.025$ N/mm^2 $\sigma_B = \sigma_0 + \sigma_{bx} + \sigma_{by} = 0.175$ N/mm^2 $\sigma_C = \sigma_0 - \sigma_{bx} + \sigma_{by} = 0.075$ N/mm^2 $\sigma_D = \sigma_0 - \sigma_{bx} - \sigma_{by} = -0.075$ N/mm^2	— 1M
	Additional load $P = \sigma . A$ $P = 0.075\times12\times10^6 = 900\times10^3$ N	— 1M
	stress at each corner after additional load $\sigma_A = \sigma_0 + \sigma_{bx} - \sigma_{by} = 0.1$ N/mm^2 $\sigma_B = \sigma_0 + \sigma_{bx} + \sigma_{by} = 0.25$ N/mm^2 $\sigma_C = \sigma_0 - \sigma_{bx} + \sigma_{by} = 0.15$ N/mm^2 $\sigma_D = \sigma_0 - \sigma_{bx} - \sigma_{by} = 0$ N/mm^2	— 1M

Q. 5		
	$D_{AB} = 100\,mm \quad L_{AB} = 200\,mm$	
	$D_{BC} = 50\,mm \quad L_{BC} = 300\,mm$	
	$G = 8\times10^4\ N/mm^2$	
	$T = 10\ kN.m = 10\times10^3\ N.m = 10\times10^6\ N.mm$	
	Shear stress in shaft AB	
	$\tau_{AB} = \frac{T}{I_{P(AB)}} \times R_{(AB)} = \frac{T}{\frac{\pi}{32} D_{AB}^4} \times \frac{D_{AB}}{2}$	
	$\tau_{AB} = 50.93\ N/mm^2$	— 2M
	Shear stress in shaft BC	
	$\tau_{BC} = \frac{T}{I_{P(BC)}} \times R_{(BC)} = \frac{T}{\frac{\pi}{32} \times D_{BC}^4} \times \frac{D_{BC}}{2}$	
	$\tau_{BC} = 407.43\ N/mm^2$	— 2M
	Total Angle of Twist of entire shaft	
	$\theta = \theta_{AB} + \theta_{BC}$	
	$\theta = \frac{T L_{AB}}{G_{AB} I_{P(AB)}} + \frac{T L_{BC}}{G_{BC} I_{P(BC)}}$	
	$\because G = G_{AB} = G_{BC}$	
	$\theta = \frac{T}{G}\left[\frac{L_{AB}}{I_{P(AB)}} + \frac{L_{BC}}{I_{P(BC)}}\right]$	
	$\theta = 0.06366\ Rad$	— 2M
	$\theta = 0.06366 \times \frac{180}{\pi} = 3.647^\circ$	— 1M
	Twist Angle Diagram 	— 1M

Scale 1 cm = 10 N/mm²



— 3M for diagram

Major principal stress σ_1 = Length OP × Scale

$\sigma_1 = 11.1 \times 10 = 111$ N/mm² — 1M

Minor principal stress σ_2 = Length OQ × Scale

$\sigma_2 = 3.9 \times 10 = 39$ N/mm² — 1M

Direction of Major principal plane

$2\theta = \angle DEP = 45°$

$\theta = \theta_1 = 22.5$ — 1M

$\theta_2 = \theta_1 + 90° = 112.5°$

Max. Shear Stress q_{max} = Length EF × Scale

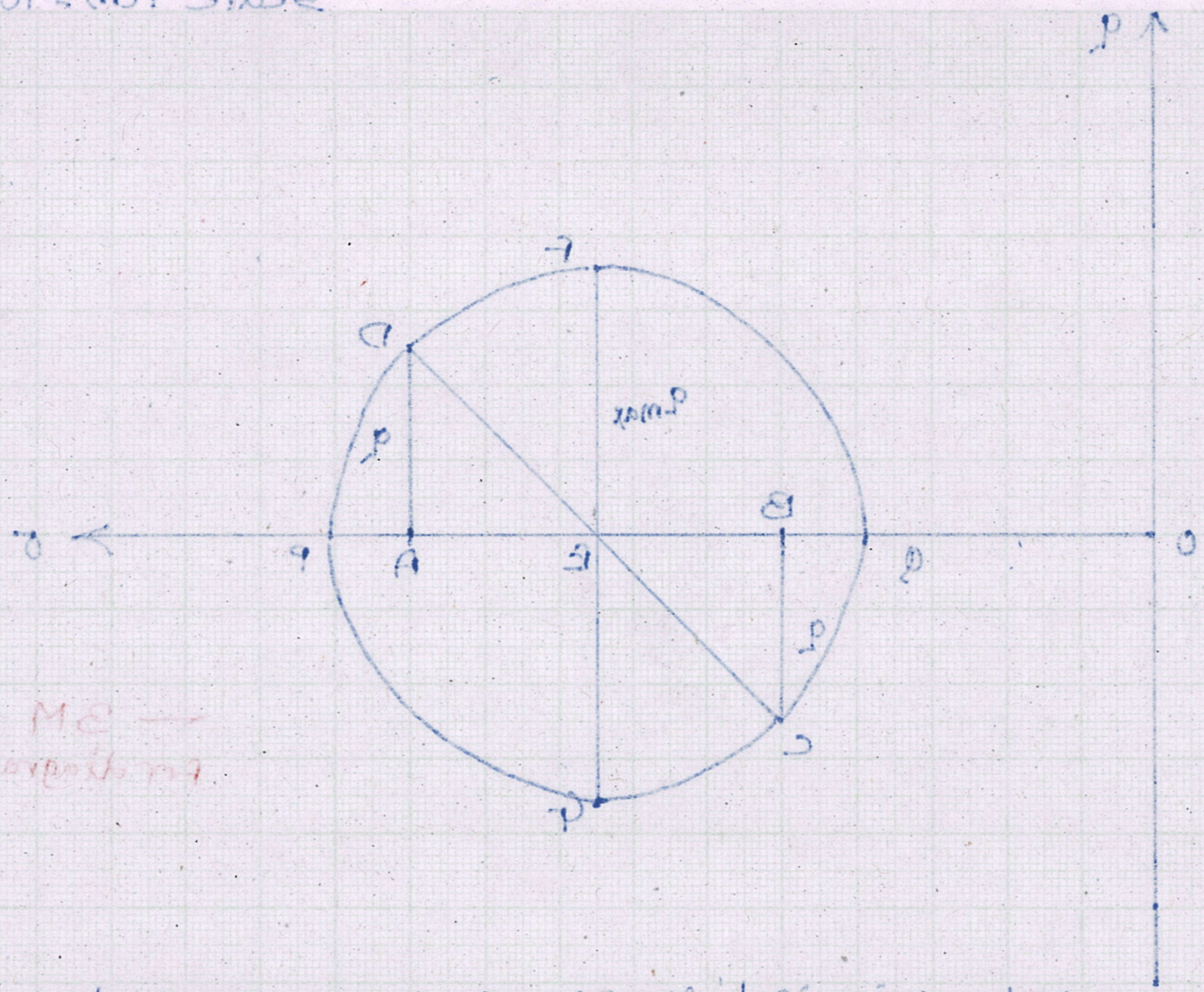
$q_{max} = 3.6 \times 10 = 36$ N/mm² — 1M

Normal Stress on Max Shear Plane

σ_n = Length OE × Scale

$\sigma_n = 7.5 \times 10 = 75$ N/mm² — 1M

Scale 1cm = 10 N/mm²



3M for diagram

Major Principal Stress σ_1 = Length OP x Scale

$\sigma_1 = 11.1 \times 10 = 111\ N/mm^2$ — 1M

Minor Principal Stress σ_2 = Length OQ x Scale

$\sigma_2 = 3.9 \times 10 = 39\ N/mm^2$ — 1M

Direction of Major Principal plane

$2\theta = \angle DEP = 45°$

$\theta = \theta_1 = 22.5$ — 1M

$\theta_2 = \theta_1 + 90° = 112.5°$

Max. Shear Stress q_{max} = Length EF x Scale

$q_{max} = 3.6 \times 10 = 36\ N/mm^2$ — 1M

Normal Stress on Max Shear Plane

σ_n = Length OE x Scale

$\sigma_n = 7.5 \times 10 = 75\ N/mm^2$ — 1M

Q. 6	**Solve any one of the Following.**	
a)		

b)

Alberto Castigliano in 1873 described methods to find deflections and slopes in beams and trusses as his dissertation for engineering diploma, which are known as Castigliano's theorems. This is a classic example of application of energy principles to calculating deformations. The two conditions that govern his proposition are that the body is stressed within elastic limit and the deformations are linear functions of the loads.

Castigliano's first theorem is a special case of Engesser's theorem of complementary energy with the proviso that the deformations must be linear functions of the loads. The theorem can be stated as follows:

In a linearly elastic system, the partial derivative of the total strain energy stored in a structure with respect to the displacement at a point is equal to the force at that point.

Stated mathematically, $\delta U/\delta\Delta = P$

Considering a linearly elastic material, it is clear that the strain energy and complementary energy are equal. Thus, according to Engesser's theorem of complementary energy, $U^* = \int \Delta\delta P$; $U = \int P\delta\Delta$ and the two are equal in the case of linearly elastic material.

Castigliano's second theorem is again limited to linearly elastic systems. Stated simply,

In a linearly elastic system, the partial derivative of the total strain energy stored in the structure with respect to any force gives the displacement at that point in the direction of the force.

In the above theorem, displacement can mean a translation/rotation and the force can be a load or a couple.

To prove the theorem, consider the beam loaded as shown in Fig. 10.40.

4M

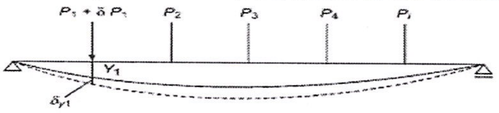


Fig. 10.40 Castigliano's theorems

$P_1, P_2, \ldots$ are the gradually applied loads acting on the beam and let $y_1, y_2, \ldots$ be the deflections under the loads. Then the total work done by the external forces is

$W_e = U = (1/2)P_1y_1 + (1/2)P_2y_2, \ldots$, where U is the strain energy stored in the beam.

Now we increase load P_1 to $P_1 + \delta P_1$ and let the additional deflections be δy_1, $\delta y_2, \ldots$, etc.

The additional work done by the loads can be expressed as

$$\delta W_e = \delta U = (1/2)\delta P_1 \ddot{a}\ y_1 + P_1\delta y_1 + P_2\delta y_2 + \ldots$$

Total strain energy U_t stored with the two applications of the loads is

$$U_t = [(1/2)P_1y_1 + (1/2)P_2y_2 + \ldots] + [(1/2)\delta P_1\delta y_1 + P_1\delta y_1 + P_2\delta y_2 + \ldots]$$

Now consider the same beam loaded simultaneously with the same loads but $P_1 + \delta P_1$ applied along with P_2, P_3 gradually. The deflection under load $P_1 + \delta P_1$ will be $y_1 + \delta y_1$, under load $P_2y_2 + \delta y_2, \ldots$, etc. The total external work done or strain energy stored can be expressed as

$$U_t = (1/2)(P_1 + \delta P_1) + (1/2)P_2(y_2 + \delta y_2) + \ldots$$

Since the strain energy must be the same due to the two methods of application of the load, the two strain energies obtained must be equal. Therefore,

$$[(1/2)P_1y_1 + (1/2)P_2y_2 + \ldots] + [(1/2)\delta P_1\delta y_1 + P_1\delta y_1 + P_2\delta y_2 + \ldots]$$
$$= (1/2)(P_1 + \delta P_1) + (1/2)P_2(y_2 + \delta y_2) + \ldots$$

Simplifying, we get

$$(1/2)P_1\delta y_1 + (1/2)P_2\delta y_2 + \ldots = (1/2)\delta P_1 y_1$$

We have neglected the term $\delta P_1 \ddot{a}\ y_1$ being a small quantity of second order. The left-hand side is the δU, the increase in strain energy calculated earlier.

We thus get $\delta U/\delta P = y_1$, the deflection under the load P_1.

Castigliano's theorem can thus be stated as follows:

The partial derivative of the total strain energy of any structure, which is linearly elastic, with respect to any of the applied forces is equal to the displacement of the point of application of that force in the direction of the force.

We have derived the theorem for a set of point loads. This is equally applicable for the moments applied on the structure. The displacement in this case will be the rotation of the point of application of the moment in the direction of the applied moment.

Castigliano's theorem gives deflection $y = \delta U/\delta P$. When we consider deflection due to bending, $U = \int M^2_x dx/2EI$, the integration being done over the segment or span length.

Therefore, $y = \delta[\int M^2_x dx/2EI]/\delta P$. The partial derivative is taken with respect to the load P, which gives the deflection in the direction of P. It will be convenient to take the derivative within the integral sign before integrating. Thus,

4M

$$y = \int 2M(\delta M_x/\delta P)dx/2EI = S\ [M(\delta M/\delta P)dx/EI]$$

If the deflection is required at a point where no load is acting, a fictitious load can be assumed to be acting. This load is set to zero after differentiation but before integration.

Course Coordinator