SET II

G .S. Mandal's

# MAHARASHTRA INSTITUTE OF TECHNOLOGY, AURANGABAD

(An Autonomous Institute)

Name of the Examination: Second Year B.Tech (Computer Science & Engineering) Feb/Mar 2023

Name of the Course : **Discrete Mathematics and Graph Theory**

Course Code : CSE 202

Name of the Expert : Mr. Rahul Mapari

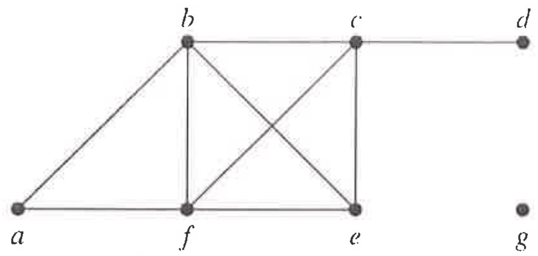
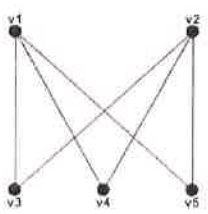
Designation : Assistant Professor

Department : Computer Science and Engineering

Contact Number : 9028084446

| S.No. | Sub Q.No. | | Marks |
|---|---|---|---|
| 1 | a | Define Proposition<br>Answer:<br>Is a statement which is either true or false but not both at the same time. | 2 Marks |
| | b | Represent following set with set builder notation:<br>" A is a set of all integers greater than 10"<br>Answer:<br>A={x\| x>10, x is integer} | 2 marks |
| | c | Let P={a,b,c} and Q={1,2,3,4}, and f: P-> Q such that f={(a,2),(b,1),(c,1)}<br>Find the domain, and range of function.<br>Answer:<br>D(R )={a,b,c}<br>Rn(R)={1,2} | 2 Marks |

**SET II**

| | | | |
|---|---|---|---|
| | d | What is combination?<br>Answer:<br>In mathematics, a combination is a way of selecting items from a collection where the order of selection does not matter. Suppose we have a set of three numbers P, Q and R. Then in how many ways we can select two numbers from each set, is defined by combination. | 2 marks |
| | e | Give the degree of each vertex for the below graph<br><br>Answer:<br>a=2,b=4,c=4,d=1,e=3,f=4,g=0 | 2 Marks |
| | f | Define Semi Group.<br>Answer:<br>An algebraic structure (S,*) which has associative property is called semigroup. | 2 marks |
| | G | Let, P={1,2,3,4}, Q={a,b,c} and<br>R={(1,a)(2,b)(3,c)(4,c)(4,b)}<br>Find Complement of R<br>Answer:<br>Complement of R={(1,b),(1,c),(2,a),(2,c),(3,a),(3,b),(4,a)} | 2 marks |
| | H | Draw $K_{2,3}$ and $K_{1,5}$<br>Answer:<br> | 2 marks |

**SET II**

**2**

Use mathematical induction to show that

$$1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$$

for all nonnegative integers $n$.

*Solution:* Let $P(n)$ be the proposition that $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$ for the integer $n$.

BASIS STEP: $P(0)$ is true because $2^0 = 1 = 2^1 - 1$. This completes the basis step.

INDUCTIVE STEP: For the inductive hypothesis, we assume that $P(k)$ is true for an arbitrary nonnegative integer $k$. That is, we assume that

$$1 + 2 + 2^2 + \cdots + 2^k = 2^{k+1} - 1.$$

To carry out the inductive step using this assumption, we must show that when we assume that $P(k)$ is true, then $P(k+1)$ is also true. That is, we must show that

$$1 + 2 + 2^2 + \cdots + 2^k + 2^{k+1} = 2^{(k+1)+1} - 1 = 2^{k+2} - 1$$

assuming the inductive hypothesis $P(k)$. Under the assumption of $P(k)$, we see that

$$\begin{aligned} 1 + 2 + 2^2 + \cdots + 2^k + 2^{k+1} &= (1 + 2 + 2^2 + \cdots + 2^k) + 2^{k+1} \\ &= (2^{k+1} - 1) + 2^{k+1} \\ &= 2 \cdot 2^{k+1} - 1 \\ &= 2^{k+2} - 1. \end{aligned}$$

Note that we used the inductive hypothesis in the second equation in this string of equalities to replace $1 + 2 + 2^2 + \cdots + 2^k$ by $2^{k-1} - 1$. We have completed the inductive step.

Because we have completed the basis step and the inductive step, by mathematical induction we know that $P(n)$ is true for all nonnegative integers $n$. That is, $1 + 2 + \cdots + 2^n = 2^{n+1} - 1$ for all nonnegative integers $n$. ◀

8 marks

**OR**

Demonstration That p ∨ (q ∧ r) and (p ∨ q) ∧ (p ∨ r) Are Logically Equivalent.

Answer:

| $p$ | $q$ | $r$ | $q \wedge r$ | $p \vee (q \wedge r)$ | $p \vee q$ | $p \vee r$ | $(p \vee q) \wedge (p \vee r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | F | T | T | T | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | T | T | T |
| F | T | T | T | T | T | T | T |
| F | T | F | F | F | T | F | F |
| F | F | T | F | F | F | T | F |
| F | F | F | F | F | F | F | F |

**3**

A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French, and Russian, how many students have taken a course in all three languages?

SET II

<table>
<tr><td></td><td></td><td>

Answer:

Let S be the set of students who have taken a course in Spanish, F the set of students
who have taken a course in French, and R the set of students who have taken a course in Russian.
Then

$|S| = 1232, \quad |F| = 879, \quad |R| = 114,$
$|S \cap F| = 103, \ |S \cap R| = 23, \ |F \cap R| = 14,$

and

$|S \cup F \cup R| = 2092.$

When we insert these quantities into the equation

$|S \cup F \cup R| = |S| + |F| + |R| - |S \cap F| - |S \cap R| - |F \cap R| + |S \cap F \cap R|$

we obtain

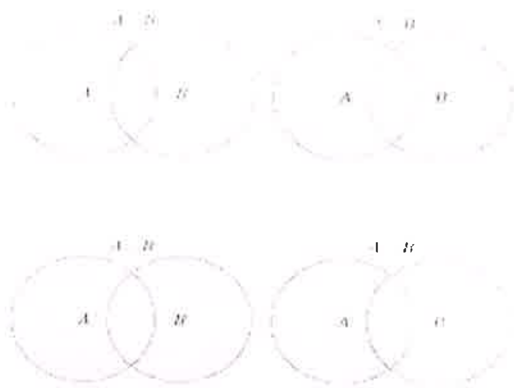
$2092 = 1232 + 879 + 114 - 103 - 23 - 14 + |S \cap F \cap R|.$

We now solve for $|S \cap F \cap R|$. We find that $|S \cap F \cap R| = 7$. Therefore, there are seven students who have taken courses in Spanish, French, and Russian. This is illustrated in Figure 4. ◄

**OR**

Draw the Venn diagrams for all set operations

Answer:



</td><td></td></tr>
<tr><td>4</td><td></td><td>

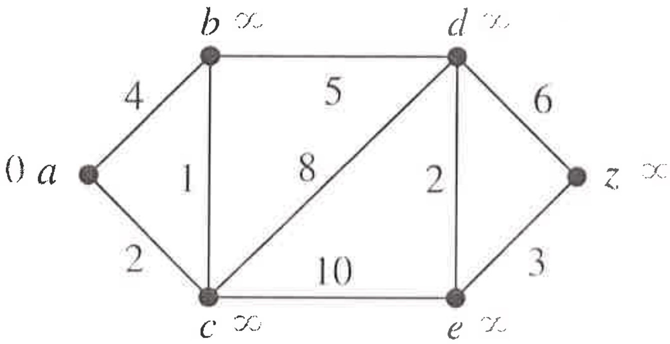
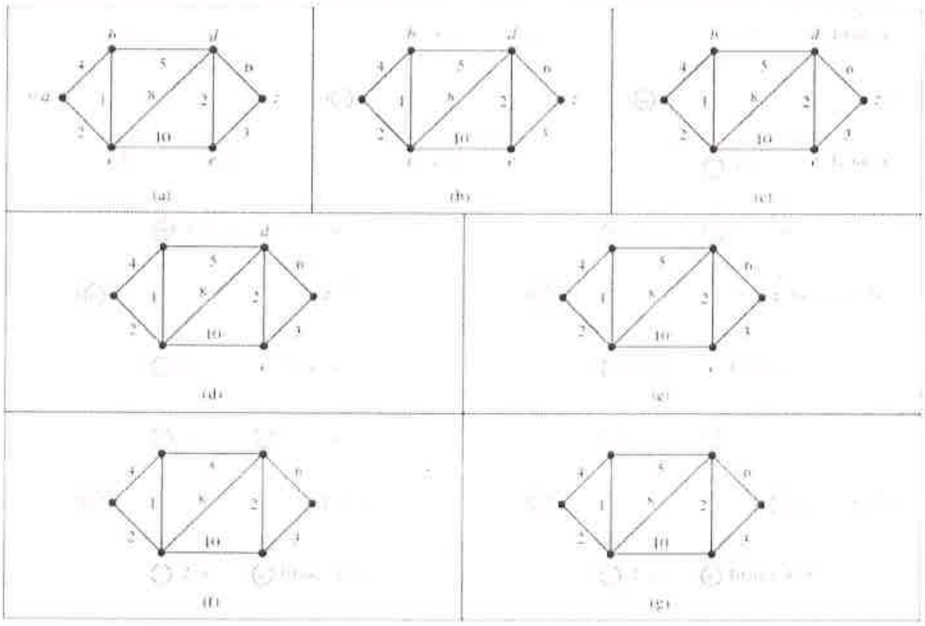
Let, A={a,b,c,d}, R={(a,a),(b,b),(c,c),(d,d),(b,a),(d,c)}, determine whether R is an equivalence relation.
Answer:

The given relation is reflexive, symmetric and transitive.

So it is a equivalence relation.

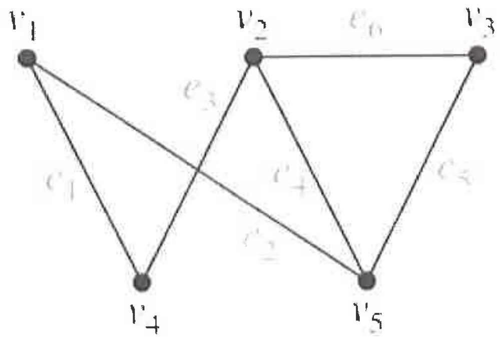
**OR**
Let A={1,2,3}, R={(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)}
Determine whether R is partial order relation or not.

</td><td>8Marks</td></tr>
</table>

| | | | |
|---|---|---|---|
| | | Answer:<br>The given relation is reflexive, antisymmetric and transitive.<br>So it is a partial order relation. | |
| 5 | | Use Dijkstra's algorithm to find the length of a shortest path between the vertices a and z in the weighted graph displayed in Figure<br><br>Answer:<br><br>**OR** | 8<br>Marks |

SET II

Represent the graph shown in Figure with an incidence matrix




Answer:

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ |
|---|---|---|---|---|---|---|
| $v_1$ | 1 | 1 | 0 | 0 | 0 | 0 |
| $v_2$ | 0 | 0 | 1 | 1 | 0 | 1 |
| $v_3$ | 0 | 0 | 0 | 0 | 1 | 1 |
| $v_4$ | 1 | 0 | 1 | 0 | 0 | 0 |
| $v_5$ | 0 | 1 | 0 | 1 | 1 | 0 |

6

Explain the product rule and sum rule with example.

8 Marks

Answer:

The **Rule of Sum** and **Rule of Product** are used to decompose difficult counting problems into simple problems.

- **The Rule of Sum** – If a sequence of tasks T1,T2,…,Tmcan be done in w1,w2,…wm ways respectively (the condition is that no tasks can be performed simultaneously), then the number of ways to do one of these tasks is w1+w2+⋯+wm. If we consider two tasks A and B which are disjoint (i.e. A∩B=∅), then mathematically |A∪B|=|A|+|B|
- **The Rule of Product** – If a sequence of tasks T1,T2,…,Tman be done in w1,w2,…wm ways respectively and every task arrives after the occurrence of the previous task, then there are w1×w2×⋯×wm ways to perform the tasks. Mathematically, if a task B arrives after a task A, then |A×B|=|A|×|B||

**Example**

**Question** – A boy lives at X and wants to go to School at Z. From his home X he has to first reach Y and then Y to Z. He may go X to Y by either 3 bus routes or 2 train routes. From there, he can either choose 4 bus routes or 5 train routes to reach Z. How many ways are there to go from X to Z?

**Solution** – From X to Y, he can go in 3+2=53+2=5 ways (Rule of Sum). Thereafter, he can go Y to Z in 4+5=94+5=9 ways (Rule of Sum). Hence from X to Z he can go in 5×9=455×9=45 ways (Rule of Product).

**OR**

Prove that the structure (Q,+,.) is a Field.

Answer:

The (Q,+) has following properties,

a. Closure
b. Associativity
c. Identity element
d. Inverse
e. Commutative

The (Q,.) has following properties,

a. Closure
b. Associativity
c. Identity element
d. Inverse
e. Commutative

So, the given ring is ring of unity as it is having multiplicative identity and it also has cumutative property.

So the given ring is Commutative ring of unity.

And every element in Q has a multiplicative inverse, so the given ring is field